Single Server Retrial Queueing System with Partial Breakdown by Computational Method

Muthu Ganapathi Subramanian*1, G. Ayyappan2, Gopal Sekar3

1Department of Mathematics, Kanchi Mamunivar Centre for Post Graduate Studies,Pondicherry, India

2Department of Mathematics, Pondicherry Engineering College, Pondicherry, India

3Department of Mathematics, Tagore arts College, Pondicherry, India

*1csamgs1964@gmail.com; 2ayyappanpec@hotmail.com; 3gopsek28@yahoo.co.in

Abstract

A single server retrial queueing with partial breakdown and repair is taken into considerations, in which customers arrive in a Poisson process with arrival rate λ. The time interval between partial breakdowns of server follows an exponential distribution with parameter α and the time interval between repairs of server follows an exponential distribution with parameter β. The server provides service with two different rates that are normal service rate $\mu 1$ and lesser service rate $\mu 2$ during a partial breakdown. If the server is free at the time of a primary call arrival, the arriving call begins to be served immediately as the server and customer leaves the system after service completion. Otherwise, if the server is busy then arriving customer goes to orbit and becomes a source of repeated calls. A pool of sources of repeated calls may be viewed as a sort of queue. Each such source produces a Poisson process of repeated calls with intensity σ. If an incoming repeated call finds the server free, it is served and leaves the system after service, while the source producing this repeated call disappears. If there is a partial breakdown of a server during the service (active breakdown), the server does the service with lesser service rate. It is assumed that the access from orbit to the service facility is governed by the classical retrial policy. This model is solved by using Direct Truncated Method. Numerical study have been done for analysis of Mean Number of Customers in the Orbit (MNCO), Truncation level (OCUT), Probabilities of server free in normal period and partial breakdown period, Probabilities of server busy in normal service period and in partial breakdown period for various values of λ, $\mu 1$, $\mu 2$, α ,β and σ in elaborate manner ; in addition, various particular cases of this model have been discussed.

Keywords

Single Server ; Direct Truncation Method; Orbit; Classical Retrial Policy ; Breakdown and Repair

Introduction

This Queueing systems in which arriving customers

Finding all servers and waiting positions (if any) occupied may retry service after a period of time are called Retrial queues. Detailed surveys on retrial queues and bibliographical information have been obtained from Artalejo [1999a, 1999b, 2010], Artalejo and Gomez-Corral [2008], Falin [1990], Falin and Templeton [1997], Yang and Templeton [1987]. Because of the complexity of the retrial queueing models, it is generally difficult to obtain analytic results. As there are a great number of numerical and available approximations methods, in this paper more emphasis will be put on the solutions using Direct Truncation method [2009a, 2009b, 2010a, 2010b, 2011a, 2011b].

Retrial queues with unreliable servers have been studied by Kulkarni and Choi [1990], Yang and Li [1994], Aissani [1993], Aissani and Artalejo [1998]. Ayyappan et al [2009a, 2009b, 2010a] have studied the unreliable server in priority services and Erlang type services for retrial queueing system. Muthu Ganapathi Subramanian et al [2011b] have investigated the unreliable servers for Multi server retrial queueing system. Kalidass and et al [2012] have studied the working breakdown for single server queueing system. In this paper, the working breakdown has been extended as partial breakdown of the server for single server Retrial queueing system.

Frequently, we confront many queueing situations, in which the server fails partially and provides service for customers with lesser rates instead of stopping services.

Model Description

A single server Retrial queueing has been taken into account with partial breakdown and repair of server, in which customers arrive in a Poisson process with arrival rate λ. The time interval between partial breakdowns of server follows an exponential distribution with parameter α and the time interval between repairs of server follows an exponential

distribution with parameter **β**. The server provides service with two different rates that are normal service rate μ_1 and lesser service rate μ_2 during partial breakdown. If the server is free at the time of a primary call arrival, the arriving call begins to be served immediately as the server and customer leave the system after service completion. Otherwise, if the server is busy then arriving customer goes to orbit and becomes a source of repeated calls. A pool of sources of repeated calls may be viewed as a sort of queue, each of which produces a Poisson process of repeated calls with intensity **σ.** If an incoming repeated call finds the server free, it is served and leaves the system after service, while the source producing this repeated call disappears. If there is a partial breakdown of a server during the service (active breakdown), the server does the service with lesser service rate and after completion of the service the customer leaves the system and the server continues the service with lesser service rate until it is repaired to the original rate.

Retrial Policy

It is assumed that the access from the orbit to the service facility follows an exponential distribution with rate nσ which may depend on the current number n, (n ≥ 0) the number of customers in the orbit. That is, the probability of repeated attempt during the interval (t, $t + \Delta t$), given that there are n customers in the orbit at time t, is nσ Δt, which is called the **classical retrial policy**.

Description of Random Process

Let N(t) be the random variable which represents the number of customers in orbit at time t and C(t) be the random variable which represents the server status at time t. The random process is described as

$$\{ < N(t), C(t) > / N(t) = 0,1,2,3,\ldots;\ C(t) = 0,1,2,3\}.$$

C(t) = 0 if the server is idle at time t during the normal service period

C(t) = 1 if the server is busy at time t during the normal service period

C(t) = 2 if the server is idle during the partial breakdown period at time t

C(t) = 3 if the server is busy during the partial breakdown period at time t

The process can be formulated as a continuous Markov chain with state space is

$$\{(u, v) / u = 0,1,2,3,\ldots;\ v = 0,1,2,3\}$$

The infinitesimal generator matrix **Q** for this model is given below

$$\mathbf{Q} = \begin{pmatrix} A_{00} & A_0 & O & O & O & \cdots \\ A_{10} & A_{11} & A_0 & O & O & \cdots \\ O & A_{21} & A_{22} & A_0 & O & \cdots \\ O & O & A_{32} & A_{33} & A_0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

The matrices described in the Infinitesimal generator matrix **Q** can be obtained from the following infinitesimal transition rates of process **X** as follows

Let $S_1 = -(\lambda+\mu_1+\alpha)$, $S_2 = -(\lambda+\mu_2+\beta)$, $S_3 = -(\lambda+i\sigma)$

$$q_{(0,j)(l,m)} = \begin{cases} \lambda & if \quad (l,m)=(0,1) \quad for \quad j=0 \\ -\lambda & if \quad (l,m)=(0,0) \quad for \quad j=0 \\ \lambda & if \quad (l,m)=(1,1) \quad for \quad j=1 \\ \mu_1 & if \quad (l,m)=(0,0) \quad for \quad j=1 \\ \alpha & if \quad (l,m)=(0,3) \quad for \quad j=1 \\ S_1 & if \quad (l,m)=(0,1) \quad for \quad j=1 \\ \lambda & if \quad (l,m)=(0,3) \quad for \quad j=2 \\ -\lambda & if \quad (l,m)=(0,2) \quad for \quad j=2 \\ \lambda & if \quad (l,m)=(1,3) \quad for \quad j=3 \\ \mu_2 & if \quad (l,m)=(0,2) \quad for \quad j=3 \\ \beta & if \quad (l,m)=(0,1) \quad for \quad j=3 \\ S_2 & if \quad (l,m)=(0,3) \quad for \quad j=3 \end{cases}$$

For i = 1, 2, 3,...

$$q_{(i,j)(l,m)} = \begin{cases} \lambda & if \quad (l,m)=(i,1) \quad for \quad j=0 \\ i\sigma & if \quad (l,m)=(i-1,1) \quad for \quad j=0 \\ S_3 & if \quad (l,m)=(i,0) \quad for \quad j=0 \\ \lambda & if \quad (l,m)=(i+1,1) \quad for \quad j=1 \\ \mu_1 & if \quad (l,m)=(i,0) \quad for \quad j=1 \\ \alpha & if \quad (l,m)=(i,3) \quad for \quad j=1 \\ S_1 & if \quad (l,m)=(i,1) \quad for \quad j=1 \\ \lambda & if \quad (l,m)=(i,3) \quad for \quad j=2 \\ i\sigma & if \quad (l,m)=(i-1,3) \quad for \quad j=2 \\ S_3 & if \quad (l,m)=(i,2) \quad for \quad j=2 \\ \lambda & if \quad (l,m)=(i+1,3) \quad for \quad j=3 \\ \mu_2 & if \quad (l,m)=(i,2) \quad for \quad j=3 \\ \beta & if \quad (l,m)=(i,1) \quad for \quad j=3 \\ S_2 & if \quad (l,m)=(i,3) \quad for \quad j=3 \end{cases}$$

If the capacity of the orbit is M then

$$q_{(M,j)(l,m)} = \begin{cases} \lambda & if \quad (l,m)=(M,1) \quad for \quad j=0 \\ M\sigma & if \quad (l,m)=(M-1,1) \quad for \quad j=0 \\ -(\lambda+M\sigma) & if \quad (l,m)=(M,0) \quad for \quad j=0 \\ \mu_1 & if \quad (l,m)=(M,0) \quad for \quad j=1 \\ \alpha & if \quad (l,m)=(M,3) \quad for \quad j=1 \\ -(\mu_1+\alpha) & if \quad (l,m)=(M,1) \quad for \quad j=1 \\ \lambda & if \quad (l,m)=(M,3) \quad for \quad j=2 \\ M\sigma & if \quad (l,m)=(M-1,3) \quad for \quad j=2 \\ -(\lambda+M\sigma) & if \quad (l,m)=(M,2) \quad for \quad j=2 \\ \mu_2 & if \quad (l,m)=(M,2) \quad for \quad j=3 \\ \beta & if \quad (l,m)=(M,1) \quad for \quad j=3 \\ -(\mu_2+\beta) & if \quad (l,m)=(M,3) \quad for \quad j=3 \end{cases}$$

Description of Computational Method

Retrial queueing models can be solved

computationally by the following techniques.

(a) Direct Truncation Method

(b) Generalized Truncation Method

(c) Truncation Method using Level Dependent Quasi Birth- and –Death Process (LDQBD)

(d) Matrix Geometric Approximation.

Direct Truncation Method

Let **X** be the steady-state probability vector of **Q,** partitioned as **X** = (x(0), x(1), x(2),...) where **X** satisfies

$$\mathbf{XQ} = \mathbf{0} \text{ and } \mathbf{Xe} = \mathbf{1}$$

where x(i) = (P_{i0}, P_{i1}, P_{i2}, P_{i3}); i = 0, 1, 2,...

The above system of equations can be solved by means of truncating the system of equations for sufficiently large value of the number of customers in the orbit. That is, the orbit size is restricted to M such that any arriving customer finding the orbit full is considered lost. The value of M can be chosen so that the loss probability is small. Due to the intrinsic nature of the system, the only choice available for study on M is through algorithmic methods. While a number of approaches are available to determine the cut-off point, M, the one that seems to perform well is to increase M until the largest individual change in the elements of **X** for successive values is less than ε a predetermined infinitesimal value.

If M denotes the cut-off point or Truncation level, then the steady state probability vector $\mathbf{X}^{(M)}$ is partitioned as $\mathbf{X}^{(M)}$ = (x(0), x(1), x(2),..., x(M)), where $\mathbf{X}^{(M)}$ satisfies

$$\mathbf{X}^{(M)}\,\mathbf{Q}^{(M)} = \mathbf{0} \text{ and } \mathbf{X}^{(M)}\,\mathbf{e} = 1,$$

where x(i)=(P_{i0}, P_{i1}, P_{i2}, P_{i3}); i = 0, 1, 2,..., M.

The above system of equations is solved exploiting the special structure of the co-efficient matrix and using numerical method. Since there is no clear cut choice for M, the iterative process may be started by taking M = 1 and increasing it until the individual elements of **X** do not change significantly. That is, if M^* denotes the truncation point then

$$\| \mathbf{x}^{M^*}(\mathbf{i}) - \mathbf{x}^{M^*-1}(\mathbf{i}) \|_\infty < \varepsilon$$

where **ε** is an infinitesimal quantity.

Stability Condition

Theorem:

The inequality $\left(\dfrac{\lambda(\alpha+\beta)}{\beta\mu_1 + \alpha\mu_2} \right) < 1$ is the necessary and sufficient condition for system to be stable.

Special Cases

- **a.** If $\mu_2 \to 0$, then this model becomes a singer server Retrial queueing system with unreliable server.
- **b.** If $\sigma \to \infty$, then this model becomes a single server classical queueing system with partial breakdown and repair.
- **c.** If ($\alpha \to 0$) or ($\beta \to \infty$) then this model becomes a single server Retrial queueing system.
- **d.** If $\mu_1 = \mu_2$ then this model becomes a single server Retrial queueing system.

System Performance Measures

In this section, some important performance measures along with formulae and their qualitative behaviour for this queueing model have been studied. Numerical study has been carried out in very large scale to study these measures.

Defining,

P(n, 0) =Probability that there are n customers in the orbit and the server is idle during the normal service period

P(n, 1)=Probability that there are n customers in the orbit and the server is busy with a customer during the normal service period

P(n, 2)=Probability that there are n customers in the orbit and the server is idle during the partial breakdown period

P(n, 3)=Probability that there are n customers in the orbit and the server is busy during the partial breakdown period.

Various probabilities for various values of λ, μ_1, μ_2 , α, β and σ can be found from section 3 and the following system measures can be studied based on these probabilities.

1) The probability mass function of server state

P_1=Prob (the server is idle during normal period)

$$= \sum_{i=0}^{\infty} p(i,0)$$

P_2=Prob (the server is idle during partial breakdown period)

$$= \sum_{i=0}^{\infty} p(i,2)$$

P_3=Prob (the server is busy with a customer during normal service period)

$= \sum_{i=0}^{\infty} p(i,1)$

P_4=Prob (the server is busy with a customer during partial breakdown period)

$= \sum_{i=0}^{\infty} p(i,3)$

2) The probability mass function of the number of customers in the orbit

Prob (no customers in the orbit) = $\sum_{j=0}^{3} p(0,j)$

Prob (i customers in the orbit) = $\sum_{j=0}^{3} p(i,j)$

3) The mean number of customers in the orbit (MNCO)

$= \sum_{i=0}^{\infty} i(\sum_{j=0}^{3} p(i,j))$

4) The probability that the orbiting customer is blocked

Blocking Probability = $\sum_{i=1}^{\infty} [p(i,1)+p(i,3)]$

5) The probability that an arriving customer enters into service immediately

$= \sum_{i=0}^{\infty} [p(i,0)+p(i,2)]$

Numerical Analysis

The values of parameters λ, μ_1, μ_2, α, β and σ are chosen so that they satisfy the stability condition discussed in section 4. The system performance measures of this model have been done and expressed in the form of tables shown below using the steady state probability vector **X** for various parameter values.

For $\lambda = 10$, $\mu_1 = 20$, $\mu_2 = 25$, $\alpha = 10$, $\beta = 1000$ and $\sigma = 100$, the steady state probability vector is **X** = (x(0), x(1), x(2),..., x(M), where

x(0) = [0.4531, 0.2143, 0.0025, 0.0180]

x(1) = [0.0224, 0.1191, 0.0008, 0.0115]

x(2) = [0.0062, 0.0638, 0.0003, 0.0063]

x(3) = [0.0022, 0.0337, 0.0001, 0.0034]

x(4) = [0.0009, 0.0177, 0.0001, 0.0018]

x(5) = [0.0004, 0.0092, 0.0000, 0.0009]

x(6) = [0.0002, 0.0048, 0.0000, 0.0005]

x(7) = [0.0001, 0.0025, 0.0000, 0.0003]

x(8) = [0.0000, 0.0013, 0.0000, 0.0001]

x(9) = [0.0000, 0.0007, 0.0000, 0.0001]

x(10) = [0.0000, 0.0003, 0.0000, 0.0000]

x(11) = [0.0000, 0.0002, 0.0000, 0.0000]

Similarly, x (n) for n ≥ 12 can be found and it is noticed that x(n) → 0 as n → ∞ . For the numerical parameters chosen above, x (n) → 0 for n ≥ 13 and the sum of the steady state probabilities becomes 0.99999999999. In the same manner, we can find steady state probability vector **X** for all values of λ, μ_1, μ_2, α, β and σ.

Performance Measures

1) Probability mass function of number of customers in the orbit

No. of customers in the orbit	Prob	No. of customers in the orbit	Probability
0	0.6878	10	0.0003
1	0.1538	11	0.0002
2	0.0766	12	0.0001
3	0.0394	13	0.0000
4	0.0203	14	0.0000
5	0.0105	15	0.0000
6	0.0054	16	0.0000
7	0.0028	17	0.0000
8	0.0014	18	0.0000
9	0.0007	19	0.0000

2) Probability that the server is idle during the normal service period= 0.485454

3) Probability that the server is idle during the partial breakdown period= 003803

4) Probability that the server is busy during the normal service period = 0.467769

5) Probability that the server is busy during the partial breakdown period = 0.042974

6) Probability that the server is busy = 0.510743

7) Mean number of customers in the orbit = 0.639525

8) Probability that the orbiting customer is blocked = 0.278477

TABLEs 1, 2 and 3 show the impact of λ (low, medium and high) and retrial rate σ over mean number of customers in the orbit and the following can be inferred

- Mean number of customers in the orbit decreases as retrial rate σ increases.
- Mean number of customers in the orbit

increases as the growth of λ.

- This model becomes a classical queueing model with partial breakdown if σ is large.

TABLE 1 LOW ARRIVAL RATE VS MEANS NUMBER OF CUSTOMERS IN THE ORBIT FOR $\Lambda = 8$ $\mu1 = 20$, $\mu2 = 15$, $A = 10$, $B = 100$ AND VARIOUS VALUES OF Σ

σ	P_1	P_2	P_3	P_4	MNCO
10	0.5875	0.0042	0.3750	0.0333	0.8348
20	0.5876	0.0040	0.3749	0.0335	0.5589
30	0.5877	0.0039	0.3748	0.0336	0.4670
40	0.5878	0.0038	0.3747	0.0337	0.4210
50	0.5878	0.0037	0.3747	0.0337	0.3934
60	0.5879	0.0037	0.3746	0.0338	0.3750
70	0.5879	0.0036	0.3746	0.0339	0.3619
80	0.5880	0.0035	0.3746	0.0339	0.3521
90	0.5880	0.0035	0.3745	0.0340	0.3444
100	0.5881	0.0034	0.3745	0.0340	0.3383
200	0.5883	0.0032	0.3743	0.0342	0.3108
300	0.5884	0.0031	0.3742	0.0344	0.3016
400	0.5884	0.0030	0.3742	0.0344	0.2970
500	0.5884	0.0029	0.3741	0.0345	0.2943
600	0.5885	0.0029	0.3741	0.0345	0.2925
700	0.5885	0.0029	0.3741	0.0345	0.2912
800	0.5885	0.0028	0.3741	0.0346	0.2902
900	0.5885	0.0028	0.3741	0.0346	0.2894
1000	0.5885	0.0028	0.3741	0.0346	0.2888
2000	0.5886	0.0028	0.3740	0.0346	0.2861
3000	0.5886	0.0027	0.3740	0.0347	0.2851
4000	0.5886	0.0027	0.3740	0.0347	0.2847
5000	0.5886	0.0027	0.3740	0.0347	0.2844
6000	0.5886	0.0027	0.3740	0.0347	0.2842
7000	0.5886	0.0027	0.3740	0.0347	0.2841
8000	0.5886	0.0027	0.3740	0.0347	0.2840

TABLE 2 MEDIUM ARRIVAL RATE VS MEAN NUMBER OF CUSTOMERS IN THE ORBIT $\Lambda = 10$, $\mu1 = 20$, $\mu2 = 15$, $A = 10$, $B = 100$ AND VARIOUS VALUES OF Σ

σ	P_1	P_2	P_3	P_4	MNCO
10	0.4846	0.0049	0.4685	0.0420	1.5771
30	0.4849	0.0045	0.4682	0.0424	0.8824
50	0.4851	0.0042	0.4680	0.0426	0.7435
70	0.4853	0.0040	0.4679	0.0428	0.6840
90	0.4854	0.0039	0.4678	0.0429	0.6510
100	0.4855	0.0038	0.4678	0.0430	0.6395
300	0.4859	0.0032	0.4674	0.0435	0.5702
500	0.4860	0.0031	0.4673	0.0437	0.5563
700	0.4861	0.0030	0.4672	0.0437	0.5504
900	0.4861	0.0029	0.4672	0.0438	0.5471
1000	0.4861	0.0029	0.4671	0.0438	0.5460
3000	0.4862	0.0028	0.4671	0.0439	0.5391
5000	0.4863	0.0028	0.4670	0.0439	0.5377
7000	0.4863	0.0028	0.4670	0.0440	0.5371
9000	0.4863	0.0027	0.4670	0.0440	0.5368

TABLE 3 HIGH ARRIVAL RATE VS MEAN NUMBER OF CUSTOMERS IN THE ORBIT $\Lambda = 12$, $\mu1 = 20$, $\mu2 = 15$, $A = 10$, $B = 100$ AND VARIOUS Σ

σ	P_1	P_2	P_3	P_4	MNCO
10	0.3820	0.0053	0.5619	0.0508	2.8716
30	0.3824	0.0048	0.5615	0.0514	1.6071
50	0.3827	0.0044	0.5612	0.0517	1.3543
70	0.3828	0.0042	0.5610	0.0519	1.2461
90	0.3830	0.0040	0.5609	0.0521	1.1860
100	0.3831	0.0039	0.5609	0.0522	1.1650
300	0.3836	0.0032	0.5604	0.0528	1.0390
400	0.3837	0.0031	0.5603	0.0530	1.0232
500	0.3838	0.0030	0.5602	0.0530	1.0138
600	0.3838	0.0029	0.5602	0.0531	1.0075
700	0.3839	0.0029	0.5601	0.0532	1.0030
800	0.3839	0.0028	0.5601	0.0532	0.9996
900	0.3839	0.0028	0.5601	0.0532	0.9970
1000	0.3839	0.0028	0.5601	0.0532	0.9949
2000	0.3840	0.0027	0.5600	0.0533	0.9855
3000	0.3840	0.0026	0.5600	0.0534	0.9824
4000	0.3841	0.0026	0.5600	0.0534	0.9808
5000	0.3841	0.0026	0.5599	0.0534	0.9799
6000	0.3841	0.0026	0.5599	0.0534	0.9792
7000	0.3841	0.0026	0.5599	0.0534	0.9788
8000	0.3841	0.0026	0.5599	0.0534	0.9784
9000	0.3841	0.0026	0.5599	0.0534	0.9782

TABLE 4 shows the impact of α over mean number of customers in the orbit and the following can be inferred

- Mean number of customers in the orbit decreases as the decline of α.
- This model becomes a single server Retrial queueing model if $\alpha \rightarrow 0$.

TABLE 4 MEAN NUMBER OF CUSTOMERS IN THE ORBIT FOR Λ = 10, μ1 = 20, μ2 = 15, Β = 100, Σ = 100 AND VARIOUS VALUES OF Α

α	P_1	P_2	P_3	P_4	MNCO
10.000000	0.485500	0.003800	0.467800	0.043000	0.639500
5.000000	0.492500	0.002000	0.483400	0.022200	0.620200
2.500000	0.496200	0.001000	0.491500	0.011300	0.610200
1.250000	0.498100	0.000500	0.495700	0.005700	0.605100
0.625000	0.499000	0.000300	0.497900	0.002900	0.602600
0.312500	0.499500	0.000100	0.498900	0.001400	0.601300
0.156250	0.499800	0.000100	0.499500	0.000700	0.600600
0.078125	0.499900	0.000000	0.499700	0.000400	0.600300
0.039063	0.499900	0.000000	0.499900	0.000200	0.600200
0.019531	0.500000	0.000000	0.499900	0.000100	0.600100
0.009766	0.500000	0.000000	0.500000	0.000000	0.600000
0.004883	0.500000	0.000000	0.500000	0.000000	0.600000
0.002441	0.500000	0.000000	0.500000	0.000000	0.600000
0.001221	0.500000	0.000000	0.500000	0.000000	0.600000
0.000610	0.500000	0.000000	0.500000	0.000000	0.600000
0.000305	0.500000	0.000000	0.500000	0.000000	0.600000
0.000153	0.500000	0.000000	0.500000	0.000000	0.600000
0.000076	0.500000	0.000000	0.500000	0.000000	0.600000
0.000038	0.500000	0.000000	0.500000	0.000000	0.600000
0.000019	0.500000	0.000000	0.500000	0.000000	0.600000
0.000010	0.500000	0.000000	0.500000	0.000000	0.600000
0.000005	0.500000	0.000000	0.500000	0.000000	0.600000
0.000002	0.500000	0.000000	0.500000	0.000000	0.600000
0.000001	0.500000	0.000000	0.500000	0.000000	0.600000
0.000001	0.500000	0.000000	0.500000	0.000000	0.600000

Conclusion

The numerical studies show the changes in the system due to impact of retrial rate and α. The mean number of customers in the orbit decreases as retrial rate increases and in turn, it increases with the increment of arrival rate. The various special cases have been discussed. This research work will be extended further by the introduction of various vacation policies, negative arrival, second optional service and unreliable server etc.

REFERENCES

Aissani, Unreliable Queueing with Repeated Orders, *Micro Electron, Reliability 33*, No.14, 2093-2106, 1993.

Aissani and Artalejo, On the single server retrial queue subject to breakdown. *Queueing Systems 30*, 309-321,1998.

Artalejo, A Classified Bibliography of Research on Retrial Queues Progress in 1990-1999 Top 7,187-211, 1999a.

Artalejo, Accessible Bibliography on Retrial Queues, Mathematical and Computer Modelling Vol 30, 223-233, 1999b.

Artalejo and Gomez-Corral, Retrial Queueing Systems-A Computational Approach, Springer, 2008.

Artalejo, Accessible Bibliography on Retrial Queues Progress in 2000-2009, Mathematical and Computer Modelling Vol 51, 1071-1081,2010.

Ayyappan, Gopal sekar and Muthu Ganapathi Subramanian, Single Server Retrial queueing System with Breakdown and Repair under Erlang Type Services - Proceeding of International Conference of Mathematical and Computational models, PSGTECH, 57-63, Narosa Publication , India, 2009a.

Ayyappan, Muthu Ganapathi Subramanian and Gopal sekar, M/M/1 Retrial Queueing System with Breakdown and Repair of Service under non Pre-Emptive Priority service, *International Review of Pure and Applied Mathematics*, Vol. 5, No. 2, 333-353, 2009b.

Ayyappan, Gopal Sekar and Muthu Ganapathi Subramanian, M/M/1 Retrial Queueing System with Breakdown and Repair of Service under Pre-Emptive Priority Service, *International Journal of Computing and Applications*, Vol. 4, No. 2, 185-200, 2010a.

Ayyappan, Gopal Sekar and Muthu Ganapathi Subramanian, M/M/1 Retrial Queueing System with Erlang Type Service by Matrix Geometric Method, International Journal of Computer Mathematical Sciences and Applications – Vol 4, 357 – 368, 2010b.

Falin G.I, A Survey of Retrial Queues. Queueing Systems 7, No.2, 127-167, 1990.

Falin G.I and J.G.C. Templeton, Retrial Queues, Champman and Hall, London, 1997.

Gopal Sekar, Ayyappan and Muthu Ganapathi

Subramanian, Single Server Retrial Queueing System with Orbital Search under Erlang-k Service-Proceedings of the International Conference on Stochastic Modelling and Simulation, VELTECH, Chennai, 77-82, Allied Publishers, India, 2011a.

K. Kalidass and R. Kasthuri, A Queue with Working Breakdown, Computers and Industrial Engineering, 63, No. 12, 779-783,2012

Kulkarni, V.G and Choi, B.D, Retrial Queues with Server Subject to Breakdown and Repairs, *Queueing systems Vol 7, No. 2*, 191-208,1990.

Muthu Ganapathi Subramanian, Ayyappan and Gopal Sekar, M/M/c Retrial Queueing System with Breakdown and Repair of Services, Asian Journal of Mathematics and Statistics, Vol 4, No.4, 214-223, 2011b.

M.F. Neuts, Matrix Geometric Solutions in Stochastic Models-An Algorithmic Approach, The John Hopkins University Press, Baltimore, 1981.

Yang T and Templeton J.G.C, A Survey on Retrial Queues, Queueing systems, Vol 2, No. 201-233, 1987.

Yang, T and Lie, H, The M/G/1 Retrial Queues with Server Subject to Starting Failures, *Queueing Systems, Vol 16*, 83-96,1994 .

Dr. A. Muthu Ganapathi Subramanian, Associate Professor of Mathematics, Department of Mathematics, Kanchi Mamunivar Centre for Post Graduate Stuydies, Pondicherry, India. His area of interest is Retrial queueing system and has published more than 40 papers in the reputed reviewed journals.

Dr. G. Ayyappan, Associate Professor of Mathematics, Department of Mathematics, Pondicherry Engineering College, Pondicherry, India. His area of interest is Stochastic process, Probability theory and Queueing theory and has published more than 60 papers in the reputed reviewed journals.

Gopal Sekar, Associate Professor of Mathematics, Department of Mathematics, Tagore arts Collegew, Pondicherry, India. His area of interest is Retrial queueing system and has published more than 40 papers in the reputed reviewed journals.